# A Modified Particle Swarm Optimization Predicted by Velocity


Zhihua Cui
Division of System Simulation and Computer Application,Taiyuan University of Science and Technology,Shanxi,P.R.China,030024
cui_zhi_hua_7645@sohu.com

Jianchao Zeng
Division of System Simulation and Computer Application,Taiyuan University of Science and Technology,Shanxi,P.R.China,030024
zengjianchao@263.net



## ABSTRACT

In standard particle swarm optimization (PSO), the velocity only provides a position displacement contrast with the longer computational time. To avoid premature convergence, a new modified PSO is proposed in which the velocity considered as a predictor, while the position considered as a corrector.The algorithm gives some balance between global and local search capability, and results the high computational efficiency. The optimization computing of some examples is made to show the new algorithm has better global search capacity and rapid convergence rate.



## Categories and Subject Descriptors

G.3 [**Probabilistic and Statistics**]: Probabilistic algorithms(including Monte Carlo), Statistical computing

## General Terms

Algorithm, Theory

## Keywords

fitness value, particle swarm optimization, positional, velocity


## 1. INTRODUCTION

The Particle swarm optimization (PSO), introduced by Kennedy and Eberhart, has been successfully applied in many areas. To avoid premature convergence, it is pivotal how to enhance the global exploration ability of the algorithm. Generally, The parameters or structure are regulated or changed to guarantee the global optimality of convergence through balancing the global exploration and the local exploitation. In many proposed modified PSO, the velocity only provide a positional displacement. It means that only part of information of velocity are utilized and the global convergence of PSO is affected by information utilization of velocity greatly.



*GECCO'05*, June 25–29, 2005, Washington, DC, USA.
ACM 1-59593-010-8/05/0006.


## 2. MECHANISM ANALYSIS OF STANDARD PSO

Consider the update equation of the standard PSO:

$$v_{jk}(t+1) = wv_{jk}(t) + c_1 r_1 (p_{jk} - x_{jk}(t)) + c_2 r_2 (p_{gk} - x_{jk}(t)) \quad (1)$$

$$x_{jk}(t+1) = x_{jk}(t) + v_{jk}(t+1) \quad (2)$$

where $x_{jk}(t)$ and $v_{jk}(t)$ are $k^{th}$ variables representing the position and velocity at time t respectively. $0 \leq w < 1$ is an inertia weight determining how much of the particle's previous velocity is preserved, $c_1$ and $c_2$ are two positive acceleration constants, $r_1$,$r_2$ are two uniform random sequences sampled from $\sim U(0,1)$,$p_{jk}$ is the $k^{th}$ variable of personal best position found by the $j^{th}$ particle and $p_{gk}$ is the $k^{th}$ variable of best position found by the entire swarm so far.

From the mechanism of standard PSO, the constant $v_{max}$ is satisfied with

$$| v_{jk}(t) | \leq v_{max} \quad (3)$$

and the $k^{th}$ variable of velocity at time t+1 is satisfied with

$$v_{jk}(t+1) = v'_{jk}(t+1), if(| v'_{jk}(t) | < v_{max}) \quad (4)$$

$$v_{jk}(t+1) = v_{max}, if(| v'_{jk}(t) | > v_{max})$$

$$v_{jk}(t+1) = -v_{max}, if(| v'_{jk}(t) | < -v_{max})$$

where $v'_{jk}(t+1)$ is the result calculated by formula (1).

Formula (2) implies the $k^{th}$ variable of position $x_{jk}(t+1)$ at time t+1 is satisfied with

$$x_{jk}(t+1) \in \bigcup(x_{jk}(t+1), | v_{jk}(t+1) |) \subseteq \bigcup(x_{jk}(t), v_{max}) \quad (5)$$

where $\bigcup(x_{jk}(t), v_{max})$ is $v_{max}$ neighborhood of $x_{jk}(t)$.

Thus at time t, position $x_{jk}(t)$ is searching within the $v_{max}$ neighborhood, and the global search capability is decreased while $v_{max}$ decreased and the global search capability reached the maximum when $v_{max} = x_{max}$, meanwhile, the position $x_{jk}(t)$ can search within the whole space and improved the algorithm's global search capability.Test results by Y.Shi are showed that $v_{max} = x_{max}$ outperform the test functions, and Pen feng discussed the convergence condition of the standard PSO without $v_{max}$ from linear control theory. In the following part, the velocity discussed will be always assumed no limitation $v_{max}$.

From numerical calculation, if the velocity is considered as a predictor, the position is a corrector. In other word, the $k^{th}$ variable of velocity $v_{jk}(t+1)$ at time t+1 can be considered the predictor of the $k^{th}$ variable of position at time t+1, and $x_{jk}(t+1)$ calculated by (2) is the corrector. In standard PSO, the algorithm only takes the corrector $x_{jk}(t+1)$ as approximation of the $k^{th}$ variable of position at time t+1. As we know, PSO is a stochastic algorithm. Can we consider the corrector $x_{jk}(t+1)$ is better than predictor $v_{jk}(t+1)$? Of course it is not always true. It means the predictor $v_{jk}(t+1)$ may be better than corrector $x_{jk}(t+1)$ in somewhere and may not in others.

## 3. INTRODUCTION OF MPSO

To improve the global search capability, MPSO is proposed in which the fitness value of the velocity and the update equations are defined and modified respectively. The more detail can be seen as follows.

First of all, let's consider the fitness value $f(\overrightarrow{V}_j(t+1))$ of the velocity vector $\overrightarrow{V}_j(t+1) = (v_{j1}(t+1), v_{j2}(t+1), ..., v_{jn}(t+1))$ of particle j at time t+1, because of the velocity is one point within the search space, the $f(\overrightarrow{V}_j(t+1))$ is the value of fitness function at point $\overrightarrow{V}_j(t+1)$ , and $v_{jk}(t+1)$ is calculated as follows:

$$v_{jk}(t+1) = wv_{jk}(t) + c_1 r_1 (p'_{jk} - x_{jk}(t)) + c_2 r_2 (p_{gk} - x_{jk}(t)) \quad (6)$$

where $p'_{jk}$ is the $k^{th}$ personal history best position found by particle j, and defined as:

$$p'_{jk} = v_{jk}(t+1), \quad (7)$$

$$if(f(\overrightarrow{V}_j(t+1)) = max\{f(\overrightarrow{V}_j(t+1)), f(\overrightarrow{X}_j(t+1)), f(\overrightarrow{P}_j)\})$$

$$p'_{jk} = x_{jk}(t+1),$$

$$if(f(\overrightarrow{X}_j(t+1)) = max\{f(\overrightarrow{V}_j(t+1)), f(\overrightarrow{X}_j(t+1)), f(\overrightarrow{P}_j)\})$$

$$p'_{jk} = p_{jk}, otherwise$$

where $\overrightarrow{X}_j(t+1) = (x_{j1}(t+1), x_{j2}(t+1), ..., x_{jn}(t+1))$ is the position vector of particle j at time t+1, and the update equation of $x_{jk}(t+1)$ is the same as formula (2).

The algorithm can be sketched as follows:

Step1. Initiate parameters such as velocity and position vectors, fitness values, population and personal history best position, K=0;

Step2. Using formula (6) , (7) and (2) to calculate the corresponding the velocity and position vector in next time, K++;

Step3.if stop cretia is satisfied, output the swarm history best position and stop the algorithm; otherwise go step2.

## 4. SIMULATION RESULTS

The benchmark functions in this section provide a balance of unimodal and multimodal as well as easy and difficult functions. The product of the standard PSO (PSO1), PSO without limitation (PSO2) and one-population MPSO (MPSO) were tested on each function. For each experiment the simulation was run 50 times and each simulation was allowed to run for 5000 evaluations of the objective function. The benchmark function parameters that were used are listed in Table I, where d is the domain around the origin in which the PSO1, PSO2, and MPSO were initialized.

**Table 1: Function Parameters**

| Function | Dimension | d |
|---|---|---|
| Spherical | 30 | 100 |
| Schaffer | 2 | 100 |
| Quadric | 30 | 100 |
| Griewank | 30 | 100 |
| Rosenbrock | 6 | 100 |

**Table 2: The Comparison of PSO1,PSO2,MPSO**

| Function | Algorithm | ACG | ACR |
|---|---|---|---|
| Spherical | PSO1 | 3048.63 | 100 |
| Spherical | PSO2 | 2749.78 | 100 |
| Spherical | MPSO | 186.66 | 100 |
| Schaffer | PSO1 | 1918.18 | 84 |
| Schaffer | PSO2 | 953.70 | 98 |
| Schaffer | MPSO | 853.14 | 100 |
| Quadric | PSO1 | 4726.94 | 32 |
| Quadric | PSO2 | 4638.06 | 34 |
| Quadric | MPSO | 2135.5 | 100 |
| Griewank | PSO1 | 2940.72 | 36 |
| Griewank | PSO2 | 2598.35 | 52 |
| Griewank | MPSO | 2020.62 | 100 |
| Rosenbrock | PSO1 | 4048.93 | 56 |
| Rosenbrock | PSO2 | 3405.63 | 76 |
| Rosenbrock | MPSO | 2410.7 | 100 |

In table II, ACG denotes the function evolutionary number, and AVR denotes the function convergence ratio. PSO1 is the worst algorithm between the four compared algorithms from convergence speed and ratio. Though PSO2 is faster than PSO1 within 1.88 per cent 15.88 per cent from convergence speed especially for Schaffer function nearly 50.28 per cent. By adding the velocity to explore the space and enhancing the global search capability greatly, the convergence speed MPSO1 is nearly half past of PSO1, and the convergence ratio nearly 100 per cent except for Rosenbrock function.

## 5. CONCLUSION

The author suggested the structure of a new modified PSO. From the above table, MPSO is a better algorithm than PSO from evaluation number and convergence ratio. Future research will include the foundation of more effective and widely used methods of updating equations, carrying out the non-numeric implementation of MPSO and the management of knowledge in MPSO.

## 6. ACKNOWLEDGEMENT

This work is Supported by Educational Department Science and Technology Funds.(No.204018)